URBAN POPULATION DENSITY FUNCTIONS: THE CASE OF THE BARCELONA REGION

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RESUM
L'anàlisi de la densitat urbana és utilitzada per examinar la distribució espacial de la població dins de les àrees urbanes, i és força útil per planificar els serveis públics. En aquest article, s'estudien setze formes funcionals clàssiques de la relació existent entre la densitat i la distancia en la regió metropolitana de Barcelona i els seus onze subcentres.
Paraules clau: densitat de població, forma funcional, àrea metropolitana, subcentre.

ABSTRACT
Urban density analysis is used in examining the spatial distribution of population within urban areas and is particularly useful in the planning of public services. In this paper, we study sixteen classical functional forms of the relationship between density and distance in the Barcelona Metropolitan Area and in eleven of its subcentres.
Keywords: population density, functional form, metropolitan area, subcentre.

RESUMEN
El análisis de la densidad urbana se puede utilizar para estudiar la distribución espacial de la población en las áreas urbanas y la planificación de los servicios públicos. En este articulo, estudiamos dieciséis formas funcionales clásicas de la relación entre densidad i distancia en la región metropolitana de Barcelona y sus once subcentros.
Palabras clave: densidad de población, forma funcional, área metropolitana, subcentro.

1. Introduction.

This paper presents an empirical analysis of urban population density patterns in the Barcelona Metropolitan Area (BMA) and its subcentres. The research has three objectives: (1) to survey the classical models of urban population density; (2) to apply these models to data for the BMA and eleven of its subcentres, and having selected the best functional form; (3) to analyse the results.

This research is of interest for two reasons:

(1) In Spain, population density is a significant variable in the planning of public services, including transportation, health care and education. This type of analysis is particularly suited to this purpose as it can improve understanding of the relationship between density and distance.

(2) Population distribution in the Barcelona Metropolitan Area differs from that in other European Metropolitan Regions. After two decades (1950-1970) of high rates of immigration from the rural areas of Spain, the population of the Barcelona municipality grew from 1,280,179 to 1,745,142 (an increase of just 36%). In the surrounding eleven municipalities, the population grew from 339,649 to 1,102,066 (an increase of 325%). The absence of any form of planning in these years had particular repercussions for population distribution. Thus, today, the urban structure is irregular and the population is concentrated in small areas. The suburbanization that occurred resulted in the formation of high-density centres within a large industrial area. All this has taken place in one of the most important

industrial regions in the European Union, the fifth metropolitan region by volume of industrial employment (Trullén, 1997). This is a particularly important facet of the study area because the growth in population density in the second half of the century has been somewhat atypical if compared with that recorded in other metropolitan areas in Western Europe.

In urban density analysis, the monocentric density model is a simple spatial model that relates urban population density to distance from the centre of the city, usually defined as the central business district (CBD). Such models can be considered as the classical models of urban population density, where classical is understood as a functional form of the density that includes only one exogenous variable: distance. The theoretical basis for the classical approach is derived from the work of von Thünen. His model of agricultural land use was later applied to an urban situation by Alonso (1964) and developed in mathematical form by Muth (1961, 1969) and Mills (1970, 1972)[1].

Here, we conduct two analyses. In the first of these, we consider the entire population of the BMA population in relation with the centre of the Barcelona municipality (henceforth known as the BMA case). We analyse the 133 municipalities in the BMA and their distance from the centre of Barcelona. In the second, we analyse 11 of these municipalities separately using census tracts as units and the distance to their centres (henceforth known as the case of the subcentres).

The paper is organised as follows: in section 2, the sixteen functional forms for the density-distance relation are introduced. Section 3 analyses the information available for the estimation in the cases of the Barcelona Metropolitan Area and its eleven subcentres. In section 4, our results are presented and finally, in section 5, our conclusions are discussed.

2. The functional forms.

Monocentric urban density analysis has received considerable attention from two disciplines: urban geography and regional science, where it has had both theoretical and empirical applications. The classic study undertaken by Colin Clark (1951) has generated an extensive body of literature dealing with empirical implementations for a wide range of metropolitan areas and cities, in different countries and at different times.

Quantitative geography has also sought to model urban population density (Stewart, 1947 and Newling, 1969, 1971). Here, we analyse sixteen functional forms that originate from both theoretical models and empirical observations, and we introduce a new, more general, functional form to the analysis. Some of these functions have been used in traffic planning studies, for example Tanner (1961) and Smeed (1963), while others have been employed in theoretical models of the housing market (Muth, 1969). The generalisation of the functional form and the comparison of results are the work of Casetti (1973), McDonald and Bowman, (1976), Kau and Lee, (1976a, 1976b), Zielinski (1979), Anselin and Can (1986) and Smith (1997). McDonald and Bowman estimated ten functional forms with data from sixteen cities, and they compared their results with the mean standard error, the determination coefficient, and the prediction of the total city population. Kau and Lee generalised the functional form by applying the Box-Cox technique to data from forty cities. Zielinski used the determination coefficient to evaluate ten functional forms, estimated for seven cities. Anselin and Cain compared five forms for a city, following the contrast of McKinnon, White and Davidson (1983). Reviews of this literature can be found in Thrall (1988), McDonald (1989), Smith (1997) and Wang and Zhou (1999). The functional form of urban population density is not unique and this means a selection process must be adopted in each case that is analysed.

[1] For a summary of theoretical works see Fujita (1989) or Anas, Arnott and Small (1998).

In order to simplify the study of urban population density and to make it easier to understand their results , Colin Clark (1951) proposed two general hypotheses.

(1) In all cities, excluding the business and commercial area, there are densely populated areas, which decrease in density as one moves away from the centre.

(2) In most cities, as time passes density decreases in the central areas and increases in the suburbs, thus producing a territorial expansion of the city. The first empirical evidence that population density falls with increasing distance from the city centre (assumed to be at the centre of the CBD) is credited to this work.

• Clark (1951) claims that urban population density can be correctly described by means of the negative exponential function:

$$D(x) = D_0 e^{bx} \quad (1)$$

where x is the distance to the centre measured in length units, $D(x)$ is stands the resident population density per surface unit, $D_0>0$ and $b<0$.

• Stewart (1947), a geographer, had previously suggested a linear relationship between density and distance:

$$D(x) = D_0 + bx \quad (2)$$

• Tanner (1961) and Smeed (1963) proposed two new functional forms in their studies of city traffic. Their contributions are based on two special cases of the gamma quadratic function. The Tanner model assumes:

$$D(x) = D_0 e^{cx^2} \quad (3)$$

with $D_0>0$ and $c<0$.

• In Smeed's case the relationship is

$$D(x) = D_0 x^e \quad (4)$$

with $D_0>0$ and $e<0$.

• Aynvarg (1968), a Soviet geographer, introduced a new functional form:

$$D(x) = D_0 e^{(bx)x^e} \quad (5)$$

with $D_0>0$, $b\neq 0$ and $e<0$.

Following these early contributions, Newling (1969, 1971) developed empirically two (linear and semi-log) quadratic forms of the negative exponential density function. He applied the concept of allometry to the density of cities and deduced rules for intra-urban allometric growth in the United States, showing mathematically that increasing density has a depressive effect on the rate of growth. Using the 1950 census data for forty-six Standard Metropolitan Areas (SMAs) and Urbanized Areas (UAs), Newling applied the parameter estimates developed by Muth (1961). Newling showed a strong correlation between density and rate of growth. He determined a critical density (32,000 persons per square mile), above which the rate of growth is negative and below which the rate of growth is positive. He suggested that the calculation of critical densities for other cities might be important for the field of planning and that there might be an optimum urban density.

• Newling (1969) modified the work of previous researchers with two quadratic forms of the exponential model, indicating a population density crater surrounding the central business district. The logarithmic transformation of Newling's modification produces a partial, upside-down, U-shaped curve:

$$D(x) = D_0 e^{(bx+cx^2)} \quad (6)$$

In later works, the same author (Newling, 1971) suggests the possibility of an intrinsically linear functional form through a polynomial of degree two:

$$D(x) = D_0 + bx + cx^2 \qquad (7)$$

both with $D_0>0$, $b\neq 0$ and $c<0$.

The existence of a crater of residential population density intuitively makes sense because CBDs in North American cities are for the most part non-residential. This assumption is not so clear in European cities. The difference between Clark's and Newling's model suggest a dynamic interpretation - Clark's model reflects the North American city in its early stages while Newling's portrays the city in a later developmental stage. The central density reduces over time and the peak density shifts outward from the city centre.

Research undertaken in the first two decades after the publication of Clark's study is characterised by the accumulation of additional empirical evidence supporting two models: those of Clark and Tanner. Typical of this line of research are Berry, Simmons and Tennant (1963), Latham and Yates (1970). Some years later, density gradients for employment and firms were introduced (Mills 1972; Kemper and Schemenner 1974). In all these studies, the functions are estimated by OLS, and no contrast for the econometric validation of the results is performed. This accumulation of empirical evidence points to another finding: the relationship between city size and density gradient. It has been demonstrated empirically that in the United States, smaller cities have steeper density gradients and are more compact than larger cities, whereas large Asian cities have steeper density gradients and are more compact (Berry et al. 1963). Evidence collected for cities in Europe, North America and Australia indicates that as cities grow their density falls and they become less compact. Winsborough (1965) associated the change in central density with a change in transportation technology resulting from increased ownership of automobiles over time.

- McDonald and Bowman (1976) in their work to test the usefulness of certain alternative gross population density functions for urbanised areas proposed two new functional forms:

$$D(x) = D_0 (x_R - x)^b \qquad (8)$$

with $D_0>0$, $b>0$ and where x_R is the radius of the urbanised area, and:

$$D(x) = D_0 e^{(ax + b/x)} \qquad (9)$$

with $D_0>0$, $a<0$ and $b>0$.

- The generalisation of the functional form for the density gradient can be attributed to Kau and Lee (1976). Starting from Clark's model and applying Box and Cox's (1964) technique, they proposed two general functional forms in order to describe the relationship between population density and the distance to the city centre:

$$\frac{D(x)^{\lambda} - 1}{\lambda} = \beta_1 + \beta_2 x \qquad (10)$$

and

$$\frac{D(x)^{\lambda} - 1}{\lambda} = \beta_1 + \beta_2 \frac{x^{\lambda} - 1}{\lambda} \qquad (11)$$

where β_1 and β_2 are the regression parameters; λ are the functional form parameters.

When λ =1, the density in form (10) is regressed on distance (Stewart's model). If λ approaches zero, the dependent variable is the natural logarithm of density (Clark's model). Similarly, (11) will reduce to the linear form when λ is equal to one, and to a double logarithmic form (Smeed's model) when λ approaches zero. Using data from U.S. urban areas in 1970, Kau and Lee found that λ exceeded zero in 23 out of 40 cases; the density function was between exponential and linear in almost 50% of the cases.

- Frankena (1978) proposed two functional forms, polynomial and exponential:

$$D(x) = D_0 + bx + cx^2 + dx^3 \qquad (12)$$

$$D(x) = D_0 e^{bx+cx^2+dx^3} \qquad (13)$$

In both cases his results indicated: $D_0>0$, $b<0$, $c>0$ $d<0$.

• The latest classical models of urban population density were developed by Zielinski (1979). In his work he compared a number of functions (Stewart, Clark, Tanner, Smeed, Aynvarg, Newling) and introduced two new functional forms:

$$D(x) = D_0 e^{(bx+cx^2)x^e} \qquad (14)$$

and

$$D(x) = D_0 e^{(cx^2)x^e} \qquad (15)$$

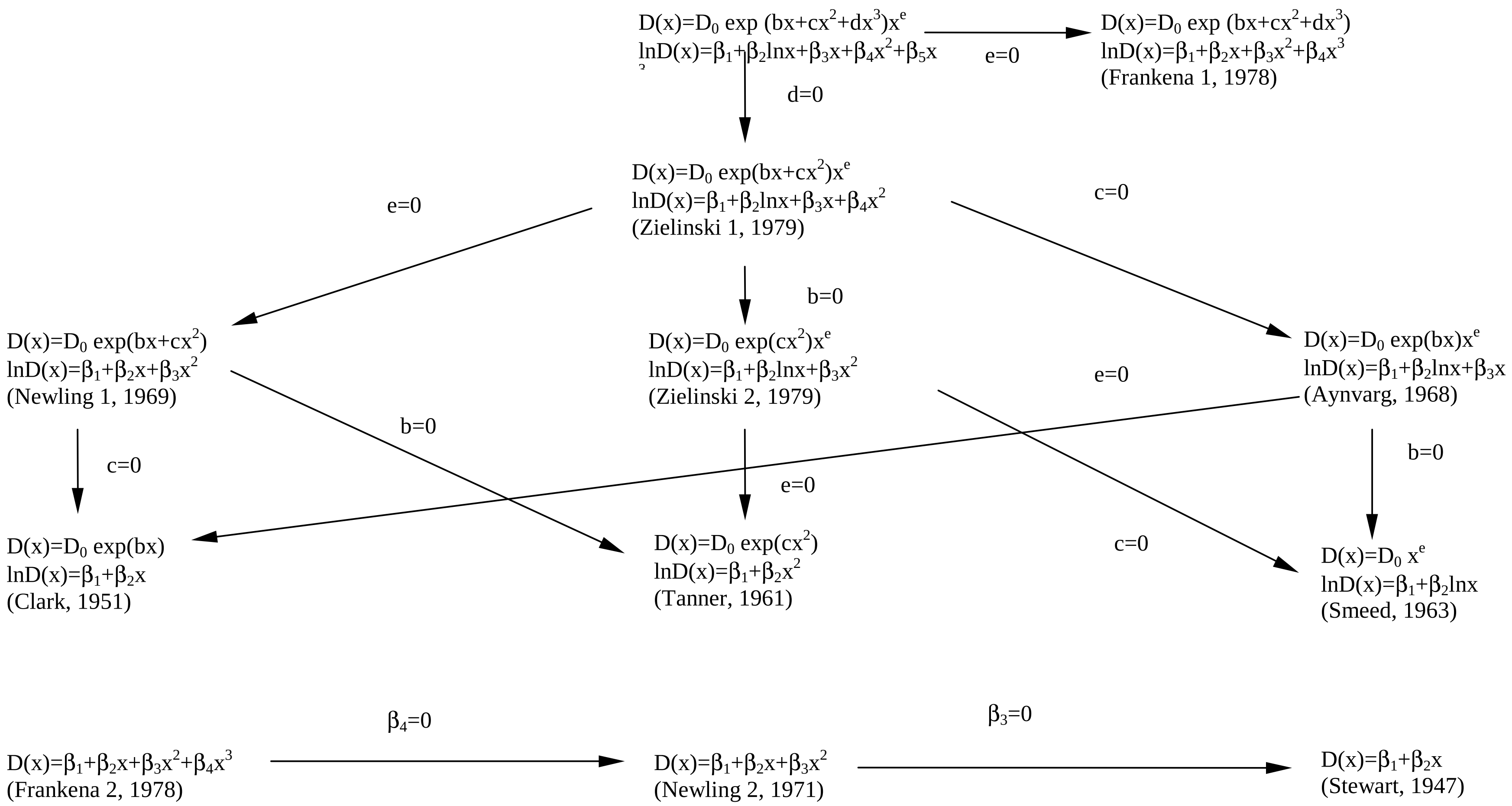
with $D_0>0$, $b>0$, $c<0$, $e<0$.

• A new, more general, functional form is introduced in the analysis conducted here. It is possible to combine the functions of Zielinski (15) and Frankena (13) and generalise the density-distance relationship. This function has never been used before:

$$D(x) = D_0 e^{(bx+cx^2+dx^3)x^e} \qquad (16)$$

In table 1 all the functional forms of the classical models of urban population density, and the relationships between the functions are shown.

TABLE 1. URBAN POPULATION DENSITY FUNCTIONAL FORMS



$D(x)=D_0(x_R - x)^b$	$D(x)=D_0 \exp(ax+b/x)$	$D(x)^{\lambda}-1/\lambda=\beta_1+\beta_2 x$	$D(x)^{\lambda}-1/\lambda=\beta_1+\beta_2(x^{\lambda}-$
$\ln D(x)=\beta_1+\beta_2 \ln(x_R - x)$	$\ln D(x)=\beta_1+\beta_2 x+\beta_3(1/x)$		
(McDonald-Bowman 1,1976)	(McDonald-Bowman 2,1976)	(Kau-Lee 1,1976)	(Kau-Lee 2,1976)

3. Data and study area.

Catalonia is a region located in the north-east of Spain (NUTS 2 in the EU classification) with 6,147,610 inhabitants (1998). Of these, 66.4% are located in the BMA (4,085,616). The Barcelona municipality has 1,266,991 inhabitants (figure 1). The BMA comprises 133 municipalities and, hence, this is the total number of units used in estimating the density functions in the BMA case. As discussed in the introduction, the main changes in population density have occurred in the municipalities surrounding that of Barcelona (see figures 2, 3 and 4). In the case of the subcentres, we use the census tracts from each municipality in making the estimation.

The definition of subcentres may vary. Giuliano and Small (1991) present an excellent discussion of this very point. Earlier studies defined subcentres and documented their presence in various ways. Some authors arbitrarily defined subcentre locations and then estimated density functions around these points, for example Bender and Hwang (1985); others used centres as defined by a regional planning agency (Greene, 1980 and Griffith, 1981); while yet another group defined subcentres as municipalities with a certain minimum size, for example Erickson (1986). Here, we employ this last definition and adopt a minimum population of 50,000 inhabitants. Thus, we find eleven municipalities. The characteristics of these eleven subcentres are summarised in table 2.

Table 2. Subcentre characteristics.

Name	Population (1998)	Area (Km^2)	Distance from the CBD of the central city (Km)	Sample size (Number of census tracts)
Hospitalet de Ll.	272,578	12.20	7.12	226
Badalona	218,725	13.56	8.89	156
Sabadell	189,404	36.11	17.05	130
Terrassa	158,063	31.77	23.77	113
S. Coloma de G.	133,138	4.95	6.89	99
Mataró	101,510	8.61	28.25	72
Cornellà de Ll.	84,927	6.05	9.02	70
S. Boi de Ll.	77,932	8.53	12.93	44
El Prat de Ll.	64,321	3.02	12.60	37
Granollers	51,873	9.67	23.14	35
Rubí	50,405	3.45	16.38	28

Source: Spanish Population Censuses.

The selection of observational units for the statistical estimation of the functional forms of population density is frequently constrained by the availability of data. Most of the more recent approaches have been based on census tract data. The census tract areas are taken as a proxy variable of the residential land. Large parts of these studies are based on the use of all the census tracts for a metropolitan area. For example, Frankena (1978) and Griffith (1981) for Toronto, Greene and Barnock (1978) for Baltimore, White (1977) for five European cities, Glickman and Ogury (1978) for 71 Japanese cities, and Alperovich (1983a, 1983b) for twenty Israeli cities.

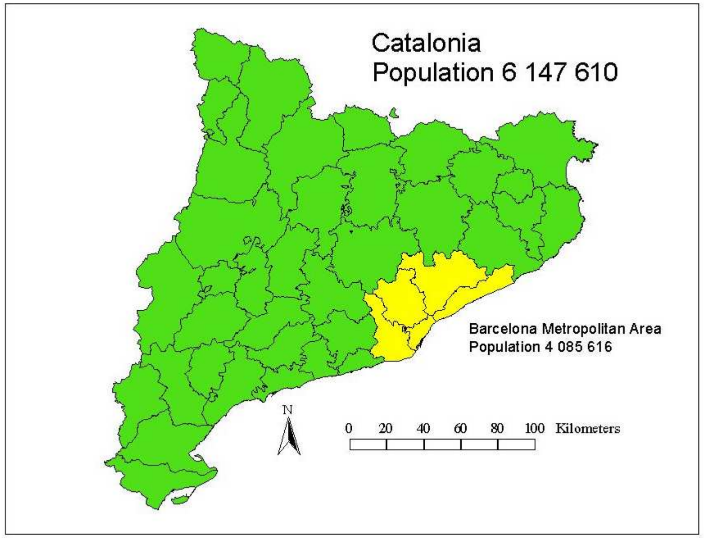


Figure 1. Catalonia and Barcelona Metropolitan Area .

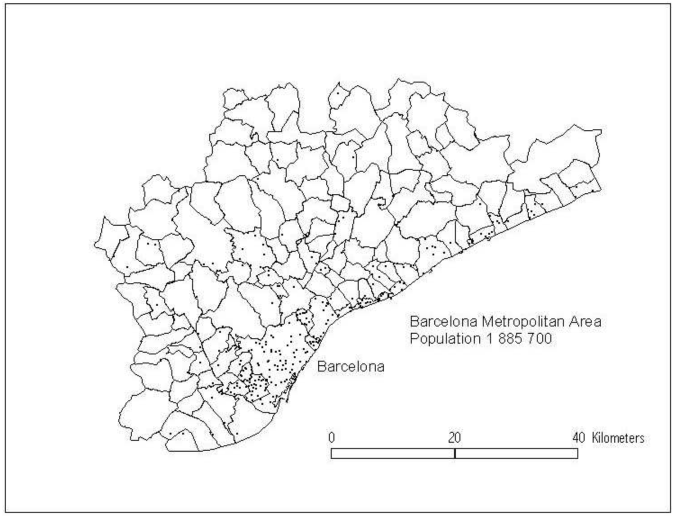


Figure 2. Dot density map of Barcelona Metropolitan Area (1950).

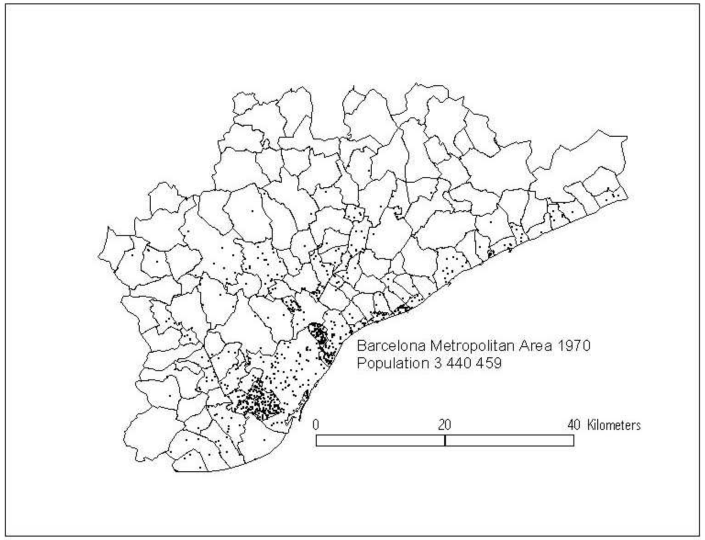


Figure 3. Dot density map of Barcelona Metropolitan Area (1970).

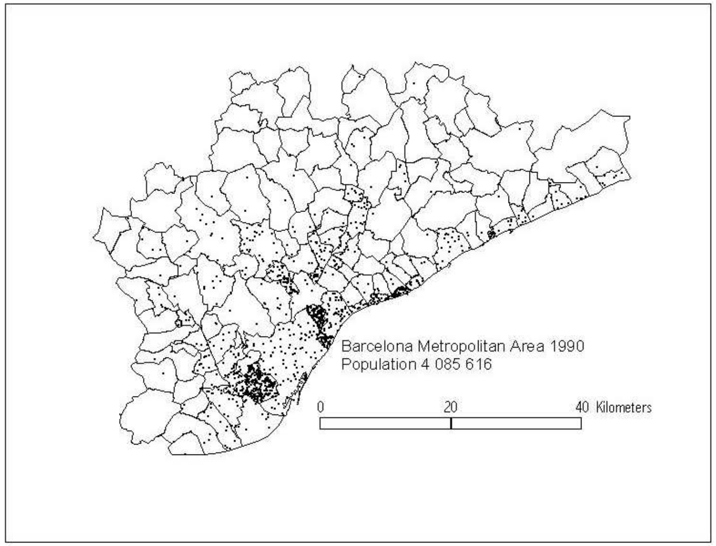


Figure 4. Dot density map of Barcelona Metropolitan Area (1990).

A number of other works, however, use a random sample of about 40 to 50 census tract observations. Examples of studies of this type include Kau and Lee (1976a, 1976b, 1977) and Kau, Lee and Chen (1983) (45 observations of 50 cities); Johnson and Kau (1980) (43 observations of 39 cities); and Anderson (1982) and Brueckner (1986) (50/70 observations for 30 cities)

Here, we use two types of data: for the BMA case, we use the municipality area; for the case of the subcentres, we use all of their respective census tracts. In the first case, the observations are a proxy of residential and industrial land and the population density can be considered a gross residential density. In the second case, the observations can be considered a proxy variable of the residential land and as such are a net residential density. We estimated the functions from the data sets of the BMA for six periods (between 1975 and 1998) and its eleven subcentres (municipalities with a population ranging from 50,000 to over 200,000) for one period (1991). The population data are taken from the Spanish Population Censuses.

The area and the distance for the BMA case were obtained with *ArcView*. In the case of the subcentres, the areas of the census tracts were measured with an electronic planimeter, while the distance to the municipality centre was measured on each city map with a ruler. In both cases we considered the straight-line distance.

The problem of determining the centre of a municipality has been studied by Alperovich (1982). Here we considered the census tract in which where the city government is located as being the centre. In the central census tract of the eleven municipalities analysed, retail and business activities were predominant. In this sense, the characteristics of the typical CBD were respected.

4. Empirical application.

One of the purposes of this paper is to select the model that provides the best functional form. Table 1 includes different models, though three types can be distinguished: linear nested, log nested and non-nested models. In the nested models our strategy was to use the F-test to select the most appropriate function for each case. We started with the most general form and then we tested whether it was possible to narrow down the specification. The strategy for the BMA case and the case of the subcentres was the same. Once we had obtained the best log and linear function for each case, we performed the Davidson-MacKinnon-White test (1983) in order to select the linear or log form.

The model selected was then compared with non-nested functions using the J-test (Davidson and Mackinnon, 1981). In some cases, this selection process is unable to select just one model, so we present different functions. We did not consider the Box-Cox transformation in this selection process[2]. In addition, applying the usual statistical tools and the selection statistic AIC, the results obtained were compared. The Goldfeld-Quandt test was performed for heteroscedasticity, the Ramsey test for linearity, and the Jarque-Bera test for disturbance normality.

Following Goldfeld-Quandt, we considered the reformulation of the statistical hypotheses for non-spherical models. In the process, several models of variance structure were tested in order to obtain efficient estimators: This involved the application of GLS. Once the variance structure had been modified, we performed the Golfeld-Quandt test again. Apart from a number of exceptional cases, the problem was solved with the GLS estimate with variance structure $V(u_i)=\sigma^2 x_i$. In those subcentres providing the poorest results, we tested other structures in an attempt to correct the problem but with negative results.

The main purpose of the heteroscedasticity analysis of this problem is to investigate urban structure stability using an econometric technique. Arnott (1980) and Brueckner (1980, 1981) have theoretically and empirically developed the growth of urban areas. Their results indicate that a growing urban area generally allows discontinuous contours of population density. This was very clear in the BMA case and the results of the Goldfeld-Quandt test that reject H_0 confirmed the presence of discontinuous contours of population density.

[2] The Box-Cox transformation (10) and (11) represents another possible way of selecting the functional form, but the Box-Cox transformation was used to analyse the simplest forms involving only one exogenous variable. Davidson and MacKinnnon (1985) proposed a new procedure for testing the null hypothesis that a regression model is linear or loglinear against the alternative of Box-Cox regression.

In table 3 we show the models selected for eight of the subcentres and for the six periods for the Metropolitan Area. Figures 5 to 13 illustrate the models selected for the BMA case (1998) and for that of the subcentres. In the cases of Terrassa, S. Boi de Ll. and El Prat de Ll. we only plotted the best AIC statistic model.

No model provided significant coefficients for Hospitalet de Ll., Cornellà de Ll and Sabadell . Two of these subcentres are located in the south of the Metropolitan Area and lie very close to Barcelona. The urban density models proposed in the literature provide better results, in general (in terms of R^2 adj.) for the BMA case. The main problem in this case is the non-normality of their disturbance, though the selection statistic AIC is more suitable.

The Newling 1 model was selected in the BMA case for all periods. The parameters for this case can be compared with the results presented by Newling (1969) for different urban areas around the world. DeBorger (1979) studied three urban areas in Belgium from 1900 to 1976 and observed that the central densities fell and the gradients flattened steadily over time. Our results for the BMA case similarly show that the gradients flattened steadily from 1975 to 1998.

The McDonald-Bowman model was selected for three of the subcentres: Badalona, S.Boi de Ll. and Granollers. The Zielinski 1 model was selected for a further three: S. Coloma de G., El Prat de Ll. and Rubí. These subcentres differ greatly in population size and are located in different zones of the BMA. The Zielinski 2, Aynvarg, Stewart and Frankena 1 models were for only one subcentre.

The value of R^2(adj.) in the subcentres indicates that the models used, with distance as the only independent variable, might be misspecified. The use of more explanatory variables in the case of net residential density might therefore be necessary. The non-normality and non-linearity problems arose in the most populated subcentres. In general, the most highly populated subcentres gave poor results for the fourteen functional forms analysed.

5. Conclusion.

This paper undertakes a survey of the classical urban population density models that have appeared since 1947. In the case of the Barcelona Metropolitan Area and its subcentres, additional explanatory variables, other than that of distance, need to be incorporated in order to model urban density correctly. Such variables should take into account certain socio-economic characteristics of the population. Additional analyses are also needed in order to examine the possible effects of central densities of subcentres, their geographic restrictions and transportation models on functional form.

Several issues require further research. First, more flexible functional forms as proposed by Anderson (1982) and Alperovich (1997) need to be estimated. Second, spatial effects need to be introduced in the classical models of urban population density, as in the studies conducted by Griffith (1981) and Anselin and Can (1986). Applying the log-normal function proposed by Parr (1985) to the BMA case is a further possibility for future research. This function is appropriate for studying the form of the regional density function, but such an approach requires larger areas.

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Annex 1

Table 3. Functional forms selected for the BMA and its subcentres.

BMA and Subcentres	*Functional Form*	*EM*	β_1	β_2	β_3	β_4	*S*	*F*	R^2*adj*	*AIC*	*J-B*	*Reset*	*GQ*
BMA (1975)	Newling 1	GLS	10.37*	-0.30*	0.0036*	---	0.65	159.21*	0.70	-111.51	61.95*	1.35	0.67
			(41.22)	(11.66)	(6.71)								
BMA (1981)	Newling 1	GLS	10.42*	-0.29*	0.0034*	---	0.65	157.17*	0.69	-111.59	59.51*	0.44	1.54
			(41.43)	(11.29)	(6.30)								
BMA (1986)	Newling 1	GLS	10.39*	-0.28*	0.0033*	---	0.64	157.11*	0.69	-117.17	55.81*	0.44	1.67
			(42.06)	(11.25)	(6.23)								
BMA (1991)	Newling 1	GLS	10.33*	-0.26*	0.0030*	---	0.62	157.23*	0.69	-127.06	50.34*	0.44	1.77
			(43.35)	(10.97)	(5.89)								
BMA (1996)	Newling 1	GLS	10.22*	-0.24*	0.0026*	---	0.60	150.05*	0.68	-137.66	62.69*	0.43	1.62
			(44.44)	(10.44)	(5.44)								
BMA (1998)	Newling 1	GLS	10.20*	-0.24*	0.0026*	---	0.59	148.45*	0.68	-139.97	56.05*	0.85	1.66
			(44.75)	(10.37)	(5.39)								
Badalona	McDonald-Bowman 1	OLS	10.36*	-0.25*	---	---	0.76	15.34*	0.15	-74.14	13.20*	13.54*	0.97
			(166.8)	(3.91)									
Terrassa	Zielinski 2	GLS	9.65*	0.53*	-0.15*	---	0.85	511.58*	0.12	-64.16	47.89*	0.11	2.07
			(79.60)	(2.30)	(3.03)								
Terrassa	Aynvarg	GLS	10.41*	1.05*	-0.92*	---	0.85	499.82*	0.10	-64.81	37.20*	1.25	2.54
			(24.8)	(2.40)	(2.59)								
S. Coloma de Gramenet	Zielinski 1	OLS	4.46* (2.05)	-2.33* (2.40)	13.10* (3.59)	-6.85* (4.69)	0.54	18.82*	0.36	-112.40	10.16*	0.09	1.71
Mataró	Stewart	OLS	22065.8	9562.1	---	---	19656.4	7.64*	0.10	1405.8	7.84*	2.93**	0.23
			(6.97)	(2.76)									
S. Boi de LL.	Newling 1	OLS	10.26* (58.9)	-0.92* (3.28)	-1.75* (3.35)	---	0.86	10.32*	0.30	-9.71	4.38	3.30*	1.68

The first column contains the name of the municipal area or subcentre, the second lists the author who introduced the functional form between density and distance, and is based on the notation used in Table 1. The third column indicates the method of estimation: OLS or GLS; columns 4, 5, 6 and 7 show the value of the coefficient and in brackets t score for the significance contrast in absolute value. In columns 8 and 9 the standard error of the regression and the value of the F statistic are shown. The adjusted determination coefficient, the selection statistic AIC, the Jarque-Bera contrast value and Ramsey's specification value appear next. In the last column, the value of the F statistic for the Golfeld-Quandt heteroscedasticity test is given. (* Significant at the 0.05 level. ** Significant at the 0.10 level)

Table 3. Functional forms selected for the BMA and its subcentres. (continuation)

BMA and Subcentres	*Functional Form*	*EM*	β_1	β_2	β_3	β_4	*S*	*F*	R^2*adj*	*AIC*	*J-B*	*Reset*	*GQ*
S. Boi de Ll.	McDonald-Bowman 1	GLS	9.91*	0.46*	---	---	0.77	1130.3*	0.24	-14.78	5.55	1.50	0.71
			(87.8)	(2.78)									
El Prat de Ll.	Frankena 1	OLS	10.28*	-1.77	-2.60*	3.94*	0.71	5.22*	0.26	-18.04	0.96	1.31	3.12
			(64.1)	(2.94)	(3.56)	(3.13)							
El Prat de LL.	Zielinski 1	OLS	26.91*	10.23*	-20.82*	4.38*	0.72	4.89*	0.25	-19.29	2.11	1.68	2.58
			(4.62)	(2.99)	(2.71)	(2.33)							
Granollers	McDonald-Bowman 1	GLS	9.26*	0.73*	--	--	0.84	685.29*	0.23	7.94	5.74	1.10	2.68
			(32.8)	(2.38)									
Rubí	Zielinski 1	OLS	1.59	-3.37**	17.16*	-8.57*	0.66	11.37*	0.54	-18.58	1.21	4.20	0.12
			(0.70)	(1.83)	(2.53)	(3.36)							

The first column contains the name of the municipal area or subcentre, the second lists the author who introduced the functional form between density and distance, and is based on the notation used in Table 1. The third column indicates the method of estimation: OLS or GLS; columns 4, 5, 6 and 7 show the value of the coefficient and in brackets t score for the significance contrast in absolute value. In columns 8 and 9 the standard error of the regression and the value of the F statistic are shown. The adjusted determination coefficient, the selection statistic AIC, the Jarque-Bera contrast value and Ramsey's specification value appear next. In the last column, the value of the F statistic for the Golfeld-Quandt heteroscedasticity test is given. (* Significant at the 0.05 level. ** Significant at the 0.10 level)

ANNEX 2

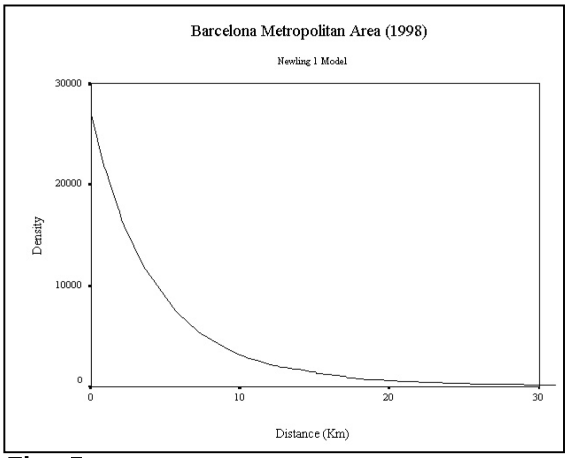


Fig. 5.

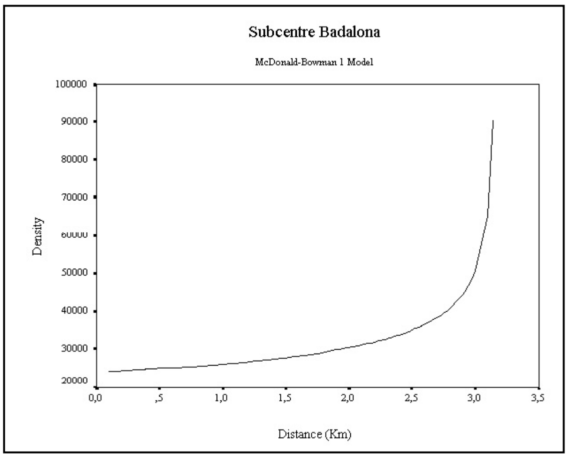
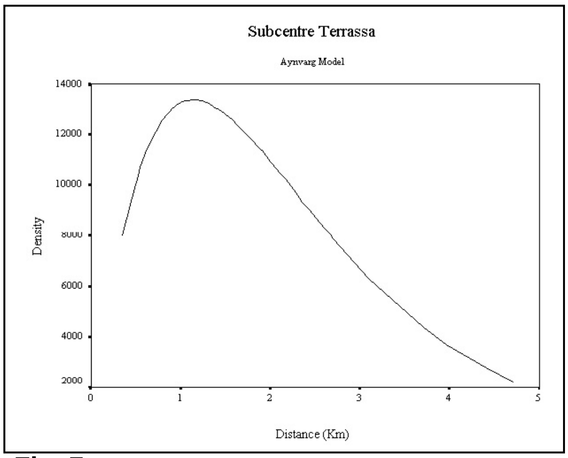


Fig. 6



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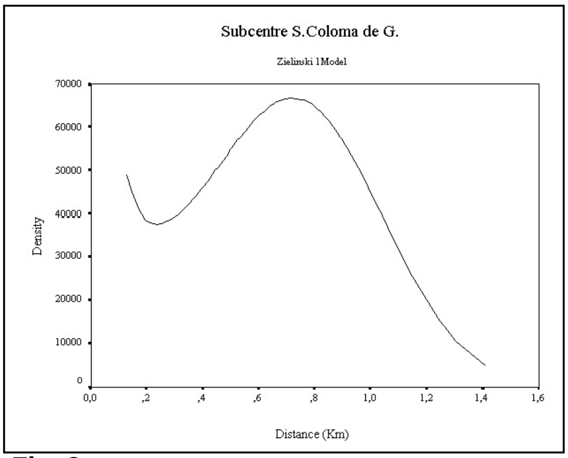

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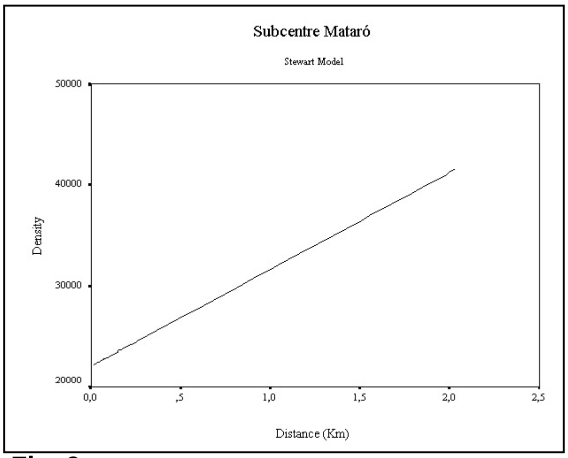

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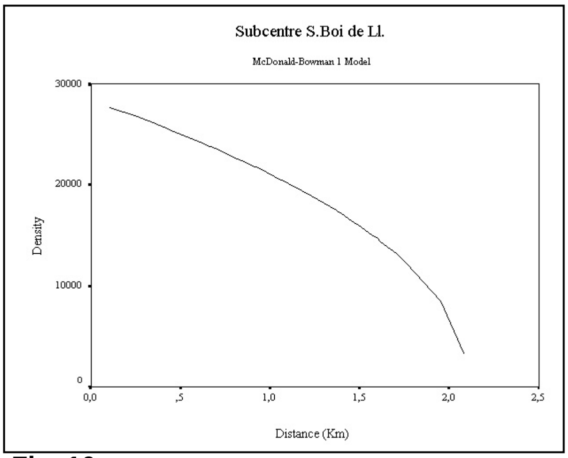

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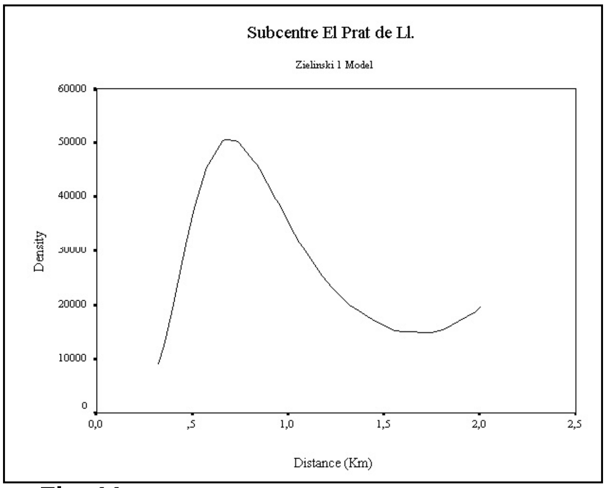

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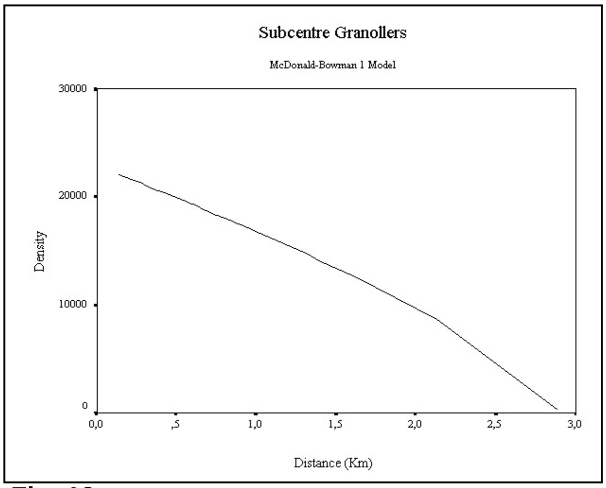

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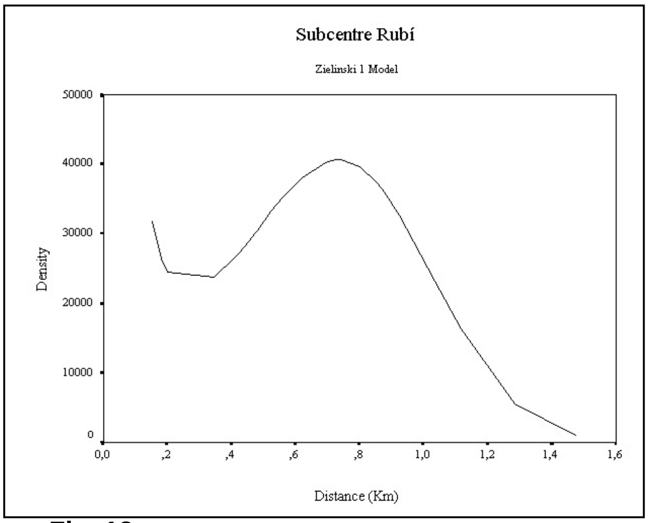

Fig. 13.